A remark on the "Theory of neutrino oscillations"

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Abstract

It is shown that the derivation of standard oscillation phase in the plane wave approximation without equal energy assumption is fraught with inconsistencies.

The literature on quantum mechanics of neutrino oscillations is vast and controversial. Therefore, the words in ref [1] "Let us briefly describe the covariant derivation of the neutrino oscillation probability in the plane wave approach" attract special attention. The derivation in ref. [1] is given in six equations on one page and is based on the substitution $t \simeq x = L$ in the phase $E_k t - p_k x$ for each of the three mass states ($k = 1, 2, 3$):

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L \quad , \quad (1)$$

where L is the distance between the source and detector of neutrinos. Our Eq.(1) reproduces the key equation of ref. [1], namely, Eq.(5).

It is further noticed in ref. [1]: "It is important to notice that Eq.(5) shows that the phase of massive neutrinos relevant for the oscillations are independent from any assumptions on the energies and momenta of different massive neutrinos, as long as the relativistic dispersion relation $E_k^2 = p_k^2 + m_k^2$ is satisfied. This is why the standard derivation of the neutrino oscillation probability gives the correct result, in spite of the unrealistic equal momenta assumption."

We would like to point out that the replacing $t \simeq x = L$ is fraught with at least two problems:

1. If one assumes that $x = t$, then the so-called space velocities $\bar{v} = x/t$ of all three neutrinos are identical. But in this case the plane wave approximation is not valid.

2. If, on the other hand, one considers the so-called kinematical velocities $v_k = p_k/E_k$, according to the plane-wave picture, then there appears a correction to the Eq.(5) of ref. [1]:

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$$x = v_k t = \frac{p_k}{E_k} t \simeq (1 - \frac{m_k^2}{2E_k^2}) t. \qquad (2)$$

Hence $t = (1 + \frac{m_k^2}{2E_k^2})x$,

$$E_k t - p_k x \simeq (E_k - p_k)L + \frac{m_k^2}{2E_k^2} L = 2\frac{m_k^2}{2E_k^2} L \qquad (3)$$

and one gets the standard formula multiplied by the notorious "factor of 2", which had been shown to be wrong. Thus, the "theory" of ref. [1] is false.

In order to derive the standard phase in the plane wave approximation for the "clockless" neutrino oscillation experiments[1], one has to assume that all three E_k are equal: $E_1 = E_2 = E_3 = E$ (see refs. [2], [3]) and replace each $p_k x$ by $EL - \frac{m_k^2}{2E} L$.

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References

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[3] H. Lipkin, hep-ph/0212093 (2002).

[1] In all neutrino oscillation experiments performed up to now time interval is not measured, only the distance between the source and detector is known. In that sense these experiments are "clockless" and imply integration over t which kills t-dependent terms.